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6. Author(s).
Robert D. Purrington

7. Performing Organization Name(s) and Address(es).
Department of Physics
Tulane University
New Orleans, LA 70118

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BOTTOM SCATTERING IN ACOUSTIC PROPAGATION MODELING

FINAL TECHNICAL REPORT

R.D. PURRINGTON

Department of Physics

Tulane University, New Orleans, LA 70118

Grant No. N00014-92-J-6008

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Abstract

The problem of propagating acoustic energy in the ocean subject to boundary roughness is considered. Both a small range-step "Monte Carlo" approach in which the acoustic signal is scattered from an ocean bottom consisting of the deterministic bathymetry with a stochastic component superimposed, and a scattering kernel method, in which the pressure field vector is analyzed at each range-step and modified by application of a scattering operator, are discussed. Results of computations using the former are presented. A higher-order, energy-conserving, finite-element parabolic equation model is used.

1.1 Introduction

The importance of incorporating interfacial roughness into acoustic propagation models is clear. This is as true with the ocean bottom as it is in the case of a wind-roughened sea surface or the under-ice canopy. In shallow water or in a downward-refracting environment propagation may be bottom-limited, and while scattering from bottom roughness is only one contribution to the loss of energy from the propagating signal, it is known to be important in some situations. The same is true in the case of arctic applications or sea surface roughness, expecially when surface interaction is strong. The roughness spectra in the three cases are clearly distinct, as are the boundary conditions,and in the case of the bottom there are many complications associated with transmission, layering, shear conversion, etc., which are either absent, or at least less important, in the other cases.

There is a large and growing literature on the problem of scattering from finite rough surfaces, growing out of problems in both the electromagnetic and acoustic domains, and especially in ocean acoustics [1]- [14]. Most of the techniques evolve from the Helmholtz-Kirchoff equation and involve different ways of characterizing the field on the boundary surface. Alternatively, the Lippmann-Schwinger equation has been employed, and occasionally other methods, such as the Green's function approach of DeSanto [15]. The various ways of attacking the problem include the Rayleigh hypothesis that the scattered field on the surface can be represented as a superposition of outgoing plane waves, even though that is known to be valid only above the highest point on the surface. The extended boundary condition method of Waterman [2] utilizes the fact that the incident and scattered waves must cancel in the region below the interface (in the case of an impenetrable boundary), and the extinction method forces the total field to vanish on the boundary (in the Dirichlet case). Much of the effort has been devoted to approximate solutions to the problem of rough surface scattering and to solving the Helmholtz equation "exactly," in the numerical sense. The latter includes the work of Thorsos, et al[16],[17], Fung and Chen [19], etc [18],[20]. Other work has dealt with the question of the stability and uniqueness of the solutions to the Helmholtz-Kirchoff integral equation, especially when it is implemented in the form of a Fredholm equation of the first kind [21]. Further studies have invoked perturbation theory or the Kirchoff approximation to reduce the problem to one that can be solved explicitly. The extensive numerical work has drawn a rather clear picture of these approximations and their ranges of validity. Finally, the recognition that surface roughness, especailly that of the bottom, is a multi-scale problem, has motivated studies of composite-roughness models. These hybrid or two-scale models, which divide the roughness spectrum into a large and small-scale part, superimposing a small-scale roughness on a slowly varying surface, suffer from the difficulty of determining the spatial wavenumber at which the separation between large- and small-scale roughness is to be made, and seem to offer more promise in the problem of scattering from the ocean surface than in the case of the ocean bottom, which is rough at "all" scales, i.e., the relevant part of the spatial roughness spectrum spans up to six orders of magnitude.

The difficulty with all of this work, from the present point of view, is that it sheds little light on the problem of incorporating rough interface scattering into propagation models, and, in particular, PE marching models. Indeed, it could be argued that the divide between these increasingly elaborate attempts to exhaust the possibilities of describing scattering from finite rough surfaces, generally in the far zone, and the problem of roughness in propagation modeling, has, if anything, grown, rather than diminished. It is, in any case, the latter problem which will be addressed here, although it will be seen that in an approach in which scattering strengths or cross sections are required (see below), they would be obtained from the work summarized above.

1.2 Incorporating Rough Bottom Scattering into a Parabolic Equation Model

The problem of interest here is the extension of these ideas to the propagation of sound in an acoustic waveguide with rough boundaries, using the parabolic equation (PE). In fact we treat only rough *bottom* scattering because of the difficulty of imposing a pressure-release boundary condition on the rough (i.e., undulating) ocean surface, but in principle this poses no essential problem for the finite-element PE model, and Tappert and Nghiem-Phu [22] and Thorsos, Ballard, and Ewart [23],[24] have shown how to deal with this problem in a split-step model.

The simplest approach to incorporating some of the effects of roughness is merely to lump all of the energy which is scattered, absorbed, and transmitted at an interface, and treat it as a loss. In a more sophisticated implementation, this loss will be angle-dependent, as in the treatment of Moore-Head, et al [25], using a split-step PE model in which the angular-dependent loss is applied in wavenumber space via an FFT. This has been generalized by Dozier, et al [26] to include diffuse scattering at the surface, using scattering strengths obtained from a deterministic model of the under-ice canopy in terms of elliptical bosses. The problem with this approach is that the FFTs required to get the plane wave spectrum at the surface (or bottom) require a depth window, hence a surface layer, with the result that the scattering takes place in a layer rather than at the surface and that the energy emerges from the surface layer down range from where it encountered the surface. This approach, which we will call the scattering kernel method, is also being investigated in detail, and will be reported upon below. At the moment, we briefly disuss its implementation.

Essentially the method consists of decomposing the field near the interface into a plane wave spectrum, i.e., carrying out an FFT into k-space, then operating on the pressure field vector in k-space with a scattering operator matrix elements are determined by calculation. The scattering operator "redirects" the energy in k-space (angle space) at each range-step. The scattering strengths which go into the scattering kernel may be determined by direct numerical integration of the Helmholtz integral equation for scattering of plane waves from a finite surface, by application of the Kirchoff approximation or a perturbation method. or by some other heuristic method. Since the scattering strength depends on the incident and final grazing angles, the roughness parameters, and properties of the two media, the storage requirements could be very great. A better solution is to use stored values or a semi-empirical expression for the scattering strength, i.e., a parameterization of the scattering cross-sections so that the process is reduced to a look-up table, or some simple closed-form expression e.g., Lambert's Law. At this point this is the preferred approach, with initial work using Lambert's Law, although we will ultimately turn to an expression obtained from the method of small perturbations which is modified to give approximately the correct dependence of the scattering strengths on frequency, incident and scattered grazing angles, and roughness parameters. Assuming that specular reflection is already correctly treated by the model, it must be extracted from the scattering strength before it is applied in k-space.

No previous attempt has been made to apply this method to the bottom, where the interface conditions are in general more complicated than at the ocean surface. Indeed, the method makes a realistic treatment of the ocean bottom (sediment, basement) difficult or impossible. But given the uncertainty principle, i.e., the need for a finite surface layer, there seems no escape from these difficulties.

One is entitled, of course, to ask whether the scattered energy of which we are speaking is important at al, i.e., whether it is important to propagate it down range (or up range as well, as in reverberation studies), rather than to merely treat it as a loss. We proceed here on the assumption that this is indeed the case, at least in some situations, although in the final analysis, it will be studies such as this which will answer that important question.

1.3 Small Range-Step or "Full-Physics" Approach

The only remaining realistic approach seems to be to directly scatter the acoustic energy from a randomly rough interface using the physics built into the model itself. Clearly the small-scale roughness has to be imposed statistically, and the essential element in this approach is the small range-step, which is chosen to insure that the model "feels" the stochastic roughness. This approach we call the small range-step or "full-physics" approach. Its implementation entails generating a randomly rough surface, which in the case of bottom scattering is to be superimposed on the deterministic bathymetry. The acoustic energy is propagated with a finite element PE model, subject to the usual interface conditions on the boundary, using a range-step which is small compared to a wavelength. Through the interface conditions, the model will reflect energy from the impedance discontinuity and the results become very like application of the Kirchoff approximation, in which the scattering is locally specular. Energy conservation, in a restricted sense, can be assured by requiring continuity of $p/\sqrt{\rho c}$ across the vertical rises of the stair-step interface [29]. The question of energy conservation has been addressed by Collins and Westwood [29] and, more generally, by Porter, Jensen, and Ferla [30].

In order to accurately sample the statistical roughness, a range-step considerably less than the acoustic wavelength is used, typically 1-10m at frequencies of 25-125 Hz. The bathymetry consists of a deterministic part, determined ultimately by the gridding of the bathymetric data, with a Gaussian zero-mean stochastic component superimposed upon it. The calculation consists in marching the finite-element PE solution out in range subject to the impedance matching conditions at the bottom, for each realization of the statistically rough bottom. The final transmission loss is the result of averaging over the N realizations of the bathymetry (where N is 25-100), each with the same statistical properties, to minimize the effect of statistical fluctuations.

Specifically, if we decompose the complex pressure field u into a coherent and an incoherent part [31]

$$u = < u > + \delta u \tag{1.1}$$

where clearly $< \delta u >= 0$, then it is easily shown that

$$< |u|^2 >= | < u > |^2 + < |\delta u|^2 > \tag{1.2}$$

where the terms on the right are the coherent (specular) and incoherent contributions to the received intensity. In terms of transmission loss,

$$TL_{tot} = TL_{coh} + TL_{inc} \tag{1.3}$$

Clearly $TL_{tot} \geq TL_{coh}$.

In the case of a smooth bottom, some energy is in general transmitted across the interface, while the rest is specularly reflected. The energy transmitted into the fluid or elastic bottom may be refracted back into the water column, it may be reflected by layered structures in the bottom, etc. If the interface is rough, however, the reflected energy will be scattered into directions other than the specular, in general smoothing out he typical pattern of convergence zones and interference effects.

1.4 The Parabolic Equation Model

The PE model used is a finite element, higher order, energy-conserving model described by Collins and Westwood [29], and by Collins [32], and written by Michael D. Collins; the development here follows that of Ref. 32. The model accurately propagates high angle energy by means of a Pade' representation of the square root operator which appears in the parabolic approximation to the Helmholtz equation:

$$\nabla^2\psi + k^2\psi = 0 \tag{1.4}$$

For two-dimensional propagation, the (r,z) equation becomes, in the far field, with the$1/\sqrt{r}$ cylindrical spreading factor removed,

$$\frac{\partial^2\Phi}{\partial r^2} + \rho\frac{\partial}{\partial z}\frac{1}{\rho}\frac{\partial\Phi}{\partial z} + k^2\Phi = 0 \tag{1.5}$$

The wavenumber $k = \omega/c$ may be complex. If Eq. (2.5) is factored into incoming and outgoing parts

$$(\frac{\partial}{\partial r} + ik_oK)(\frac{\partial}{\partial r} - ik_oK)\Phi = 0 \tag{1.6}$$

where $K = [1 + k_o^{-2}(\rho\frac{\partial}{\partial z}\frac{1}{\rho}\frac{\partial}{\partial z} + k^2 - k_o^2)]^{1/2}$.

When the square root operator in the outgoing component is expanded in a series of Pade' approximants, the deceptively simple form results:

$$\frac{\partial\Phi}{\partial r} = ik_oK\Phi \tag{1.7}$$

$$= ik_o\left[\frac{1 + \sum_j a_{2j-1,M}x}{1 + a_2j, M}\right]\Phi \tag{1.8}$$

where x is defined by $K = (1 + k_o^{-2}x)^{1/2}$. The Pade' coefficients are

$$a_{2j-1,M} = \frac{2}{2M+1} sin^2 \frac{i\pi}{2M+1} \tag{1.9}$$

$$a_{2j,M} = cos^2 \frac{i\pi}{2M+1} \tag{1.10}$$

If Eq. (2.8) is written $(1 + a_{2j,M}x)\frac{\partial \Phi}{\partial r} = (ik_o a_{2j-1,M}x)\Phi$, then it may be discretized in z by the Galerkin method. In the present case, the radial integration is carried out using a Crank-Nicholson algorithm.

In the energy conserving PE employed here, the only change is that K now has an additional term $\alpha = \sqrt{\rho c}$ and the outgoing equation now becomes

$$\frac{\partial \Phi}{\partial r} = ik_o K \Phi \tag{1.11}$$

with $K' = [1 + k_o^{-2} \left[\frac{\rho}{\alpha} \frac{\partial}{\partial z} \frac{1}{\rho} \frac{\partial}{\partial z} \alpha + k^2 - k_o^2 \right]$. For further deails, see Collins and Westwood [29] or Collins [32].

1.5 Bottom Roughness Spectra

Many studies[33], [34] have shown the usefulness of characterizing ocean bottom roughness in terms of a roughness power spectrum which exhibits a power law behavior. Such power spectra readily embody the fact that the bottom is rough over five to six orders of magnitude, from a spatial scale of on the order of 1 mm to 1 km. There is, in fact, a natural cut-off at low spatial wavenumbers, where the deterministic part of the bathymetry takes over, and at very high spatial wavenumbers when the roughness scale is very much less than the acoustic wavelength λ. This is not the place to explore the largely geophysical question of the power-law parameters, or, indeed, whether a simple power law is the best, or even an adequate, representation of the statistical properties of ocean bottom roughness. For illustrative purposes, however, we will discuss the effects of two roughness spectra, one having a k^{-5} behavior, with a roll-off at the origin [35], the other being a Gaussian. Specifically, we use

$$W_1(k) = 3\pi\ell\sigma^2/[2(1+\ell^2k^2)^{5/2}] \tag{1.12}$$

$$W_2(k) = \sigma^2\ell\frac{2}{\sqrt{\pi}}exp^{-k^2\ell^2/4} \tag{1.13}$$

Here **k** is the acoustic wavenumber, σ the rms roughness, and ℓ the correlation length. Although the choice of roughness power spectrum may be expected to be important in certain situations, this is not the case in the examples presented here. One expects that the frequency dependence of the scattering to depend on the roughness power spectrum used, since that determines the extent to which the scale of the roughness is commensurate with the acoustic wavelength.

1.6 Application to Parabolic Equation Modeling: Computations

In this work we employ a higher-order, "energy-conserving" PE code in the acoustic environment shown in Figure 1; the bottom parameters are given in Table I. The bottom roughness, which here consists of a statistical component superimposed upon a flat bottom, is sampled by using range-steps of 1 m, i.e.,λ /60 at 25 Hz and λ /20 at 75 Hz. The complex pressure fields are averaged in the manner implied by Eqs. (2.1)-(2.3), using N realizations of the statistical roughness, to obtain the total and coherent intensities, and the corresponding transmission losses. In general, the number of realizations is made as large as is computationally feasible, e.g., N=100, since the statistical fluctuations go like $\frac{1}{\sqrt{N}}$, but ordinarily one can get meaningful results for N> 25. The realizations are generated by a Fourier method described by Thorsos.[17]

1.7 Results of Numerical Studies

The implementation of the program described above is quite straightforward, though computationally intensive, and is being carried out on the CRAY YM-P at Stennis Space Center, MS. Most of the results presented are for the k^{-5} power spectrum, for which a typical realization of the bottom, with rms roughness σ of 20m and correlation length ℓ of 500m, is shown in Figure 2. In the test case described here, the acoustic field for a smooth bottom ($\sigma = 0$) bottom is as shown in Figure 3. The 75 Hz source is at 150m and the bottom is at 3000m; again, refer to Figure 1 and Table I for the acoustic environment.

The effects of bottom roughness can be immediately seen in Figures 4 and 5 in which an rms roughness of $\sigma = 20$m and correlation length $\ell = 500$m have been used (this value of ℓ is used in all the figures). In particular, comparision of the total with the coherent intensity shows that much of the reflected energy is scattered diffusely; the incoherent intensity (Figure 6) shows this fact dramatically. Taking the receiver to be at 150m, one sees that, especially between 10 and 20 km, the coherent transmission loss is lower by as much as 15 dB (e.g.,compare TL_{coh} in Figure 7 with

$\sigma = 0$ in Figure 8) and that beyond the point where the sound first interacts with the bottom (10 km) [36], TL_{coh} is always considerably less than TL_{tot}.

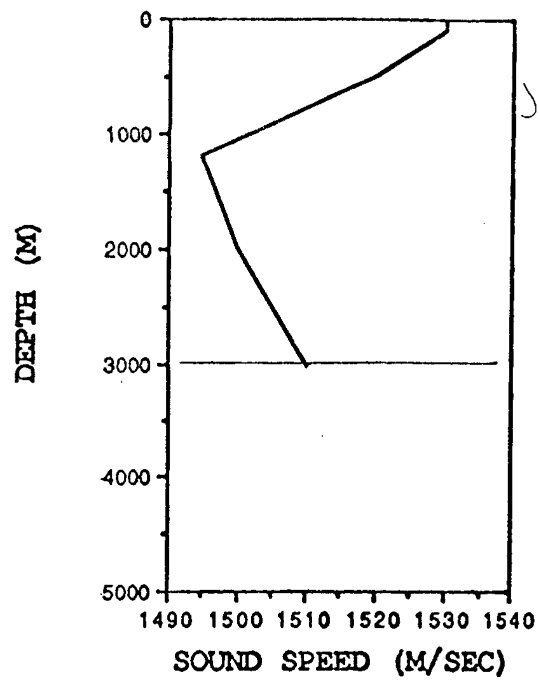

Figure 1. Sound speed profile in the water column. The sediment sound speed is 1700 m/sec. See also Table 1.

Table 1.1: Acoustic Parameters. The frequency is 25 or 75 Hz

$$c_o = 1500 msec^{-1}$$

$$\alpha_w = 0.0$$

$$\alpha_{sed}(3000) = 0.5$$

$$\alpha_{sed}(5000) = 10.0$$

$$npade' = 2$$

$$Z_S = 150m$$

The total transmission loss in Figure 8 exceeds the coherent transmission loss by as much as 20 dB in places, especially beyond 10 km, where energy from the first bottom bounce reaches the surface. The difference then settles down to 5-10 dB.

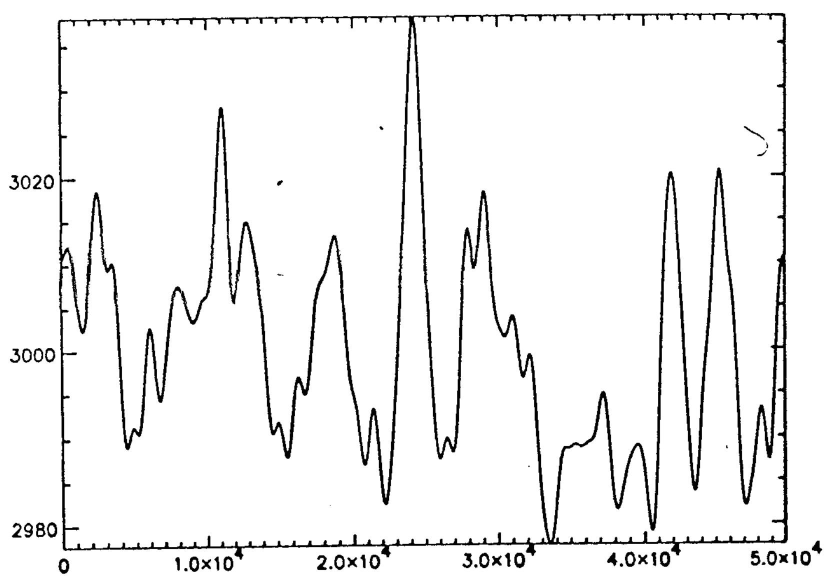

Figure 2. Example of a single realization of the bottom roughness, with $\sigma = 20m$ and $\ell = 500m$.

When the total transmission loss for $\sigma = 0$ (smooth bottom) and $\sigma = 20$m are compared (Figure 8), it is seen that beyond 10 km the transmission loss curve is somewhat smoothed-out by the roughness and intensity levels are generally lower by a few dB. Of course the effects are smaller for smaller rms roughness (Figure 5).

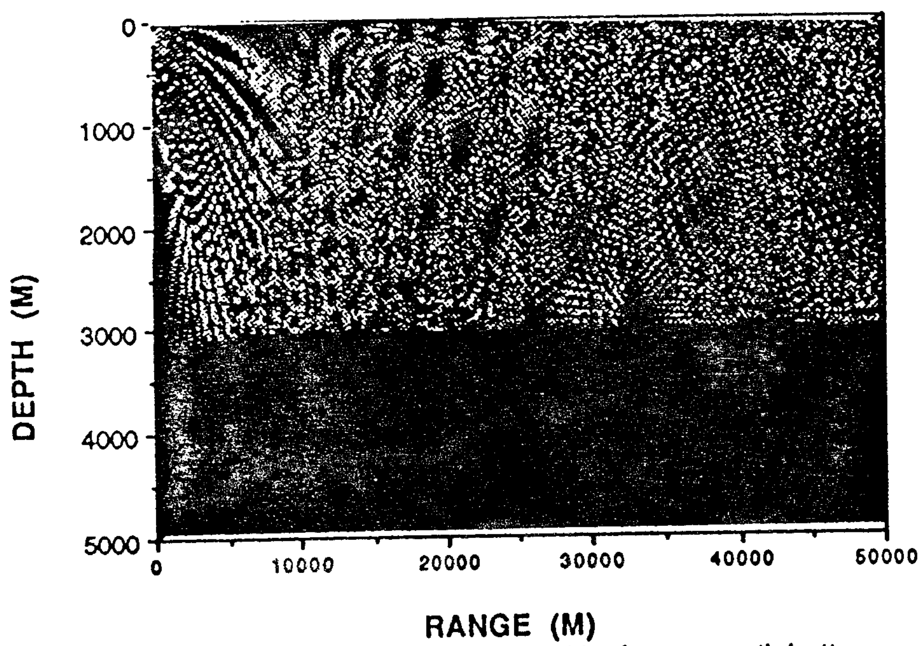

Figure 3. The acoustic intensity for a smooth bottom for the acoustic environment of Figure 1 and Table 1. The frequency is 75 Hz. Intensities are plotted from a TL of 40 dB to 100dB, from white to black.

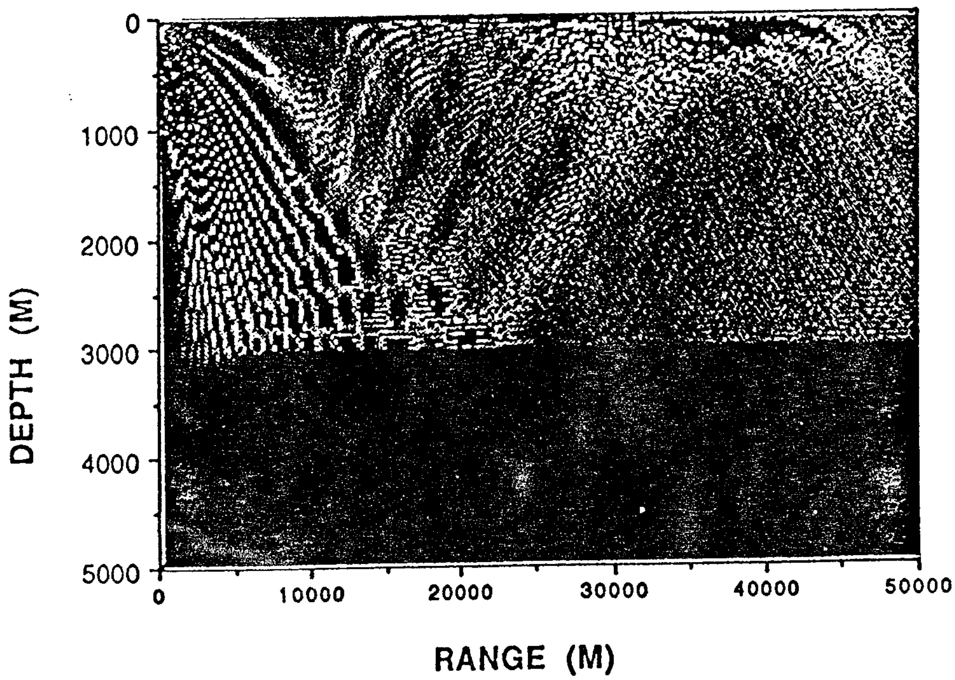

Figure 4. Total intensity $< |u|^2 >$ for rms roughness $\sigma \doteq 20m$ and correlation length $\ell = 500m$.

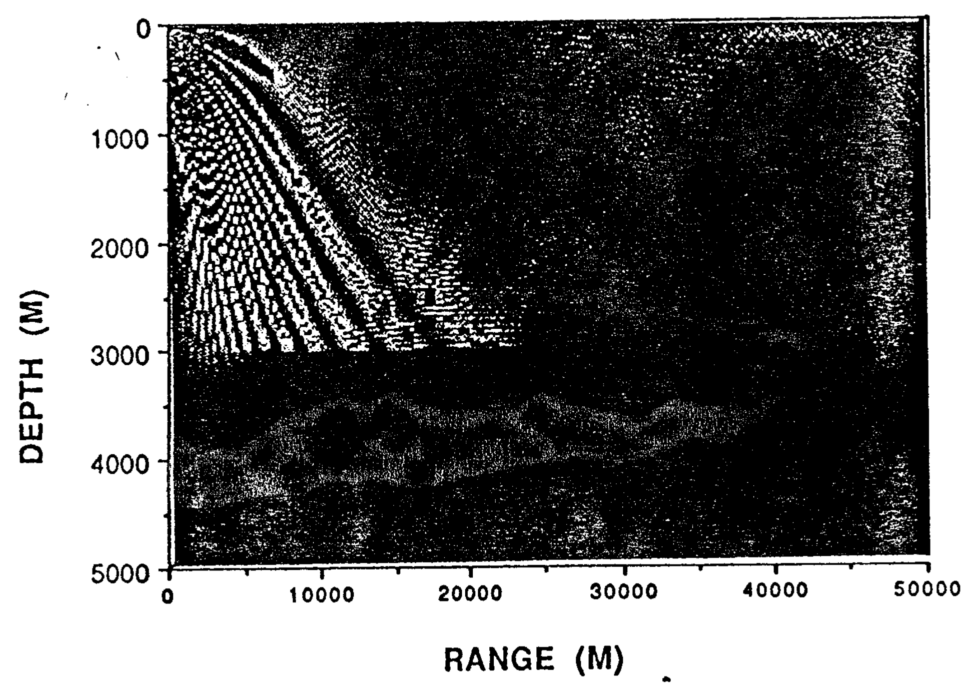

Figure 5. Coherent intensity $| < u > |^2$ for the same conditions as Figure 4.

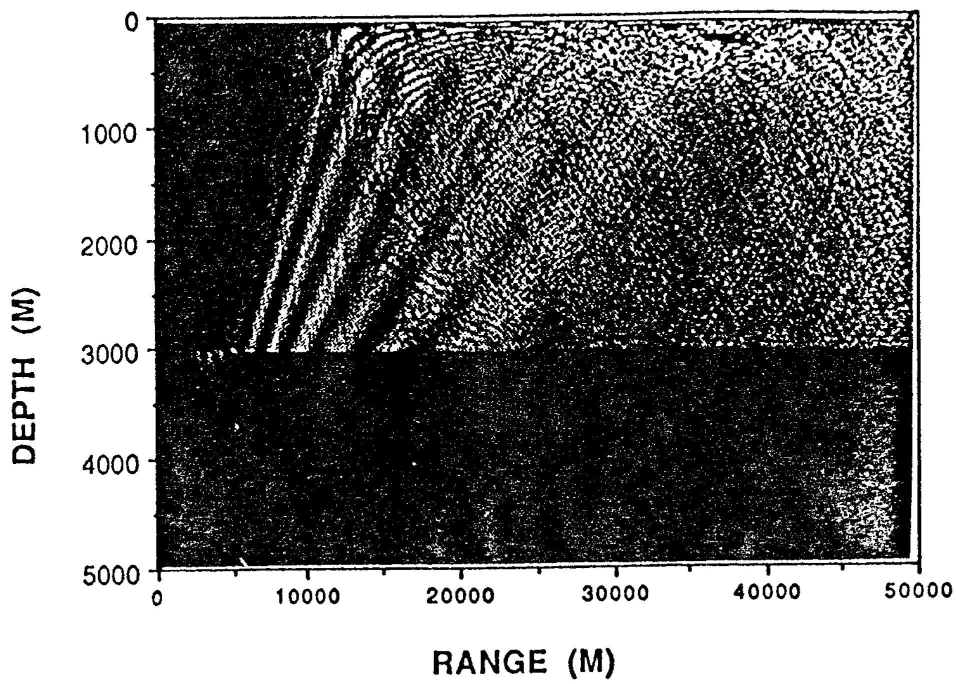

Figure 6. Incoherent intensity $< |\delta u|^2 >$

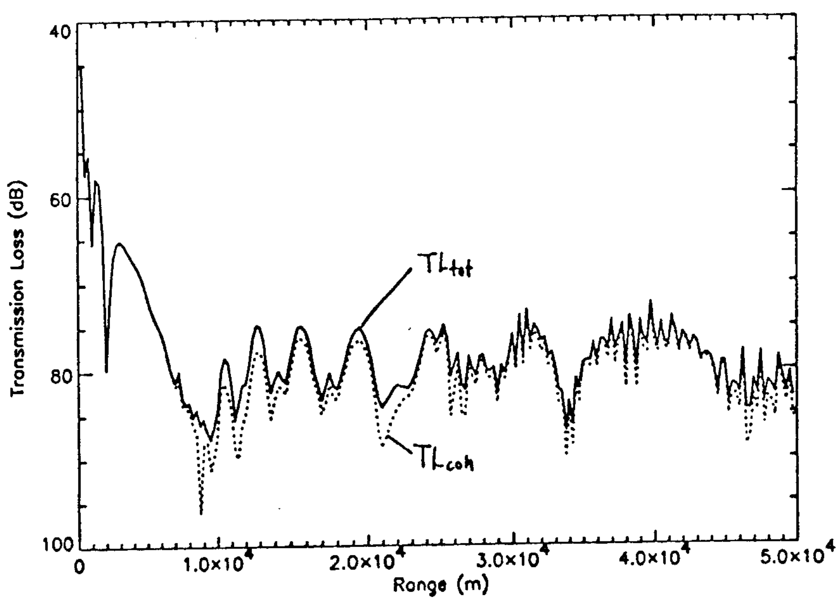

Figure 7. Total transmission loss TL_{tot} and Coherent transmission loss TL_{coh} at 75 Hz for source and receiver at 150m. In this figure, $\sigma = 20m$ and $\ell = 500m$.

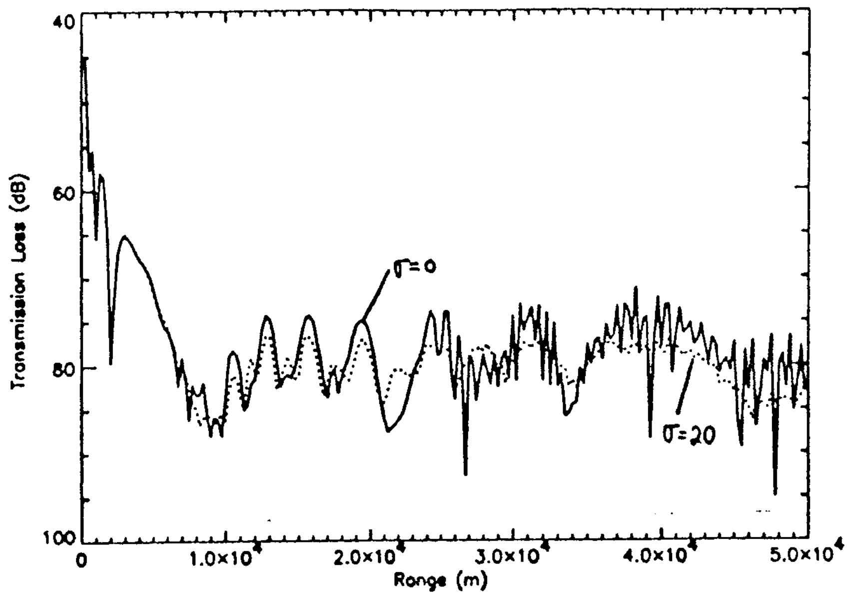

Figure 8. Total transmission loss TL_{tot} at 75 Hz for $\sigma = 0$ and $\sigma = 20m$ for source and receiver at 150m.

1.8 Frequency Dependence

Extensive calculations over the frequency range of 25-125 Mz reveal a frequency dependence to the scattering, as might be expected. Although this is difficult to illustrate, since the difference in the acoustic fields at different frequencies is not by any means due to scattering alone, it appears that the effect of an rms roughness $\sigma = 20$m is measurably greater at 100 Hz, where the acoustic wavelength is on the order of the rms roughness, than at 25 Hz, where it is much larger. We note in Figure

9 the difference between the total transmission loss for the Gaussian and k^{-5} power spectra. The differences are greatest at low grazing angles, i.e., long range, as would be expected.

1.9 Discussion

The results obtained to this point suggest that the small range-step "full-physics" approach gives a plausible description of the specular and diffuse scattering from a rough ocean bottom. Attention is now being given to the important step of providing quantitative validation of these results, both in a real acoustic environment, and theoretically. In the the latter case, one can attempt to construct an artificial isovelocity environment in which the bottom is smooth except for a scattering patch at some range R, so that approximate comparisions can be made between the PE propagated field and numerical simulations using the method of moments. It is also useful to test the results against the results of a normal-mode propagation code, e.g., KRAKEN-C [37], in which roughness is taken into account. Finally, one can attempt intermodel comparisons between the two general methods of attack described here. Extension of the small range-step approach is being made to three-dimensional propagation, where a 2-D roughness power spectrum is used.

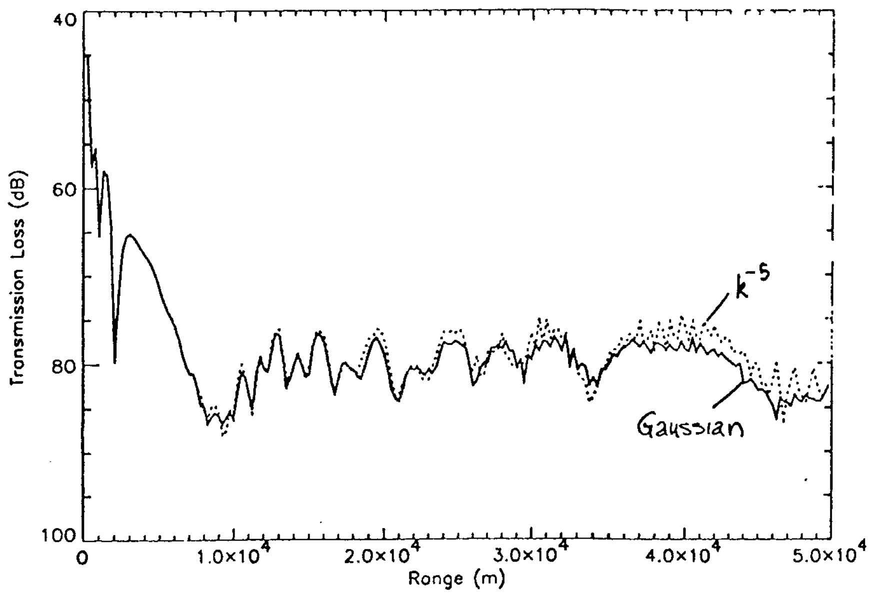

Figure 9. Dependence of the total transmission loss (TL_{tot}) on the roughness power spectrum W(k) for the same conditions as Figure 7. See Sec. 5 for details.

2.1 Scattering Kernel Method

This approach offers an interesting alternative to the "brute-force" small range-step metheod which, in principle, does not entail a significant increase in computational overhead compared to a conventional PE calculation. The scattering kernel approach consists in decomposing the pressure field near the interface (here, the bottom, but in princple the surface as well) in vertical wavenumber space and operating on the field with a scattering operator before reconstructing the field in coordinate space. Because it depends on obtaining the plane wave spectrum at the interface–in practice, in a layer adjacent to or spanning the boundary–two FFTs per range step are required. The need to keep the surface layer thin so that the method can be applied in shallow water and the desire to keep low angle rays from travelling long distances in the surface layer requires that the number of points in the FFT window be severely limited (e.g., 16, 32 points), hence the angular resolution is degraded.

If the scattering kernel is computed, in the near zone, at each range step, from the current geometry and bottom properties, then the approach is indeed computationally very intensive. If, on the other hand, some simple parameterization of the scattering from the bottom is used–the present work has thus far employed (for simplicity) only a Lambert's law representation of the scattering–then the problem is much less severe. Indeed, all of the manipulations required to obtain the scattering matrix, which depending on the approach, may involve diagonalizing an exponentiated matrix [26], can be done once, before the solution is marched out in range, or at worst, only when the deterministic bathymetry changes.

This technique was first proposed by Moore-Head, Jobst, and Holmes [25] in a split-step PE model, in order to introduce angle-dependent surface loss. The generalization by Dozier, Hanna, and Pearson [26] introduced a scattering kernel which redistributed the energy incident upon the surface, in k-space. Further work on this approach has been reported by Schneider [27]. All of the physics is in the construction of the scattering operator, which, however, can be approximated in various ways to save computational resources. These include adopting, in modified form, appropriately parameterized and scaled, the closed-form expressions for scattering in the perturbation method, or storing, in a look-up table, the results of numerical scattering experiments. This is, in any case, a matter of detail.

The method proceeds as follows [25]- [27] : Given the complex pressure field u(r,z), multiplied by a smoothing factor w(z) which helps prevent aliasing in the FFT (typically cos^2 form which tapers to 0 at the upper edge of the enlarged depth window of thickness 2d, with R, the cycle distance in the layer given by $\frac{d}{tan\theta}$,θ being the grazing angle), a discrete Fourier transform is carried out into vertical wavenumber space. The field vector in k-space is then operated upon by a scattering matrix which transforms the field according to some model of the scattering process, and finally an inverse FFT restores the pressure field. The procedure can be represented as follows:

$$\psi(r,k) = FFT[w(z)U(r,z)] \tag{2.1}$$

From which the scattered field $\phi(r,k)$ in angle-space is found:

$$\phi(r,k_1) = \sum S(k_1,k_2)\psi(r,k_2), \tag{2.2}$$

where S is the scattering operator which may also include bottom loss, etc. Finally, the pressure field u / (r,z) is recovered:

$$u'(r,z) = FFT^{-1}[\psi(r,k_2)] \tag{2.3}$$

Dozier, et al [26] worked with the intensity $|u|^2$ rather than the pressure, and showed that, properly formulated, the theory guarantees that the energy will not grow as the loss and scattering functions are applied. In this case the critical quantity is $exp(S\Delta r)$ where Δr is the range-step and S is the scattering kernel. Although S is not precisely the same operator as used above, it is again obtained from a model of the scattering process (in Dozier, et al, from a deterministic model of the under-ice surface based on cylindrical bosses); in either case any loss terms, which appear in the diagonal elements of S, represent loss per range step over the cycle distance R. In the formulation of Dozier, et al, the phase of the field is recovered from that of the original complex pressure field u(r,z).

There are some good objections, of course, to this approach. That one is on shaky ground, numerically, is obvious. By modifying the PE field at each range-step, one is in danger of violating the assumptions under which the parabolic equation was derived. More serious is the fact that the problem is no longer well-posed, and numerical instability is the likely result. At the same time, it does seem to be possible to carry out this program in practice, with great care, as the work of Moore-Head, et al, Dozier, et al, Schneider, and the author [28] seems to show. Nonetheless, it is not clear that a model which introduces an artificial bottom layer when the real bottom has its own complicated structure, shear conversion, etc., is desirable. For this reason the method is likely to have more application to the ocean surface.

Thus the usefullness of this approach remains to be established. It does offer the possibility of incorporating bottom roughness into something other than a research code. We have implemented the method in a finite-element PE code using a scattering kernel obtained from a simple Lambert's law description of the scattering. Not surprisingly the method has proved to be susceptible to aliasing and numerical instability so that it is not always easy to determine which results are real and which are numerical artifacts.

The spectral decomposition method of Gilbert and Evans [38] offers an alternative way of finding the plane wave spectrum at the interface, but has its own drawbacks, and in the end all methods for finding the PWS ultimately come up against the uncertainty principle. As mentioned earlier, the method is not well-suited for shallow water, because of the artificial bottom layer. or for low frequencies, where the layer might be less than one wavelength thick. It is certainly not well-suited to treating the bottom realistically.

3.1 Three-Dimensional Parabolic Equation Modeling

Given the success of the ensemble-averaged parabolic equation (PE) treatment of scattering from rough surfaces, it is of interest to treat three-dimensional acoustic propagation in the ocean waveguide subject to boundary roughness using the same method. The presentd study focused on the ocean bottom, for which the roughness was described using a 2-D roughness power spectrum. The final acoustic field is be the result of averaging a large number of PE runs with different realizations of the statistically rough surface. The problem is computationally intensive but naturally amenable to parallelization.

One of the central problems in ocean acoustic propagation is that of incorporating interfacial roughness into current propagation models in a realistic way. At the air-water interface this roughness will consist of the wind-roughened sea surface or the under-ice canopy. In the case of the seafloor, the roughness includes not only the water-sediment interface, and the basement, but bottom sediment layering as well. Heretofore, most of the work which has incorporated boundary roughness has done so by means of a heuristic reflection coefficient and few applications have been to parabolic equation models. Because of the somewhat greater complexity of imposing the pressure- release boundary condition on the rough ocean surface (absent ice), we shall concentrate, in this study, on the ocean bottom, and in particular the water- sediment interface.

Of the two approaches described above, both previously explored by the principal investigator [39]-[41], and employed in Parts 1 and 2 of this work, namely the ensemble-averaged, small range-step and the scattering kernel methods, it is the former that was applied to the problem of 3D acoustic propagation, using a two-dimensional roughness spectrum. This extension is motivated by the success of the previous studies and the increasing interest in true three-dimensional propagation in ocean acoustics. More generally, the importance of rough-bottom scattering lies in the need to accurately predict foward propagating acoustic signal levels using a PE model, and in the importance of active acoustic applications in which the PE field is propagated backward to the source. Although it is adequate in most situations to make the approximation of 2-D propagation, that is not true when the properties of the water column, notably the sound speed, and the character of the ocean surface or bottom vary rapidly with azimuth.

Although there are no problems in carrying out this program in principle, there are two major issues, aside from those which arise from the physics of the problem. The first is the need to optimize existing PE codes for the present purpose or to develop an original code which takes advantage of the opportunities to use parallel processing. The small range-step approach, which ensemble-averages the results of a large number of PE runs using different statistically similar realizations of the rough interface, is naturally amenable to parallelization. The second issue has to do with the 2D roughness spectrum. Although it is not a mojor goal of the proposed study to address the issue of the best representation of real bottom roughness power spectra, the effect of different representations of the statistical properties of the bottom has been studied, as well as the relation between power spectrum and acoustic frequency dependence.

This final stages of this work placed great demands upon the computational resources available to the principal investigator. The ensemble-averaged small range-step method is inherently computationally intensive, due both to the nature of the computational grid, the need to average a large number of PE runs to reduce the effects of statistical fluctuation. Its use in 3-D PE computations means execution times at least an order of magnitude greater than in the 2-D case. This turns out to be manageable, however, since in the latter case the acoustic field can be propagated out to a range of 50 km using a 1 m range-step and 25+ realizations of the bottom using 30 minutes of CRAY Y-MP8 time without parallelization.

3.2 Three-dimensional Parabolic Equation

This section describes the initial phases of the generalization of the work on scattering from rough interfaces in two dimensions, using a roughness power spectrum which is one-dimensional, to the case of full 3-D propagation. We proceed as follows, following Collins and Chin-Bing [42]:

Starting from the Helmholtz equation in three dimensions,

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} + \frac{1}{r^2}\frac{\partial^2 P}{\partial \theta^2} + \frac{1}{\rho}\frac{\partial \rho}{\partial z}\frac{\partial P}{\partial z} + K^2 P = 0 \tag{3.1}$$

Factoring into incoming and outgoing parts leads to an outgoing solution of the form

$$\frac{\partial P}{\partial r} = ik_o QP \tag{3.2}$$

where

$$Q = (1 - k_o^{-2}(K^2 - k_o^2 + \frac{\partial^2}{\partial z^2} - \frac{1}{\rho}\frac{\partial \rho}{\partial z}\frac{\partial}{\partial z} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2})^{1/2}) \tag{3.3}$$

Here K is the complex wavenumber. If the rapidly varying plane wave $e^{ik_o r}$ is factored out of P, the approximations of Ref. (4) lead to

$$\frac{\partial P}{\partial r} = \frac{2ik_o X}{(X + 4k_o^2)}P + \frac{ik_o}{2}YP \tag{3.4}$$

in which

$$X = \frac{1}{k_o^2}(K^2 - k_o^2 + \frac{\partial^2}{\partial z^2} + \frac{1}{\rho}\frac{\partial \rho}{\partial z}\frac{\partial}{\partial z}) \tag{3.5}$$

and

$$Y = \frac{1}{r^2 k_o^2}\frac{\partial^2}{\partial \theta^2} \tag{3.6}$$

The result, Eq. (15) is a wide-angle 3-D parabolic equation. In Ref. (42), this equation is solved using the method of alternating directions, Galerkin discretization, Crank-Nicholson integration in r, and centered differences in θ. The approach of Lee, et al [43] to the solution of the equivalent of Eq. (15) is to expand Q somewhat differently and to use different numerical techniques.

3.3 Numerical Implementation

The most direct route to implementation of a 3-D PE scattering/ propagation model proved to be the use of an existing three-dimensional PE code, written by Michael Collins [42]. This finite-element 3-D code was used to generate the three-dimensional complex pressure fields which are needed to carry out the ensemble averages described above, using a statistically rough seafloor (in general superimposed on whatever the deterministic bathymetry may be) obtained from a 2-D roughness power spectrum. In spite of this fact, considerable modification of the code was necessary, and it had to be made part of an emsemble-averaging, small range-step code.

Although development of the scattering/propagation code is straightforward, the demands on computer resources are naturally quite large. In the two-dimensional case, a typical calculation with 10^3 range steps involving the averaging of 50 individual runs with different realizations of the rough bottom use about 100 seconds of CRAY Y-MP8 processor time, without parallizing the code. This same calculation can be done on a dedicated IBM RS-6000/365 in less than 15 minutes, and depending on the usuage of the large scale computer, the total run time can be shorter on the RS-6000. A single 3-D run, with 10^3 range steps at 75 Hz can be performed in about 7 hours on the latter and in about 25 minutes on the CRAY, with a $\Delta\theta$ of 20° without parallelization, which can reduce the computation time by about an order of magnitude. The computations can thus be seen to be practical, but only on a supercomputer and only by parallelizing the code.

3.4 Plan of Research

The development of the 3-D scattering capability proceded as follows:

1. Development of the 3-D PE code. This consisted of modifying the existing finite-element code to output the complex pressure field instead of transmission loss. All of this development, and testing, took place on a dedicated RS-6000, a RISC cluster, and on a Convex machine at Tulane University.

2) The development of an ensemble-averaging code which calls the 3-D program, as in the 2-D case.

3) A choice of 2-D roughness power spectra was made, based on the extensive literature on seafloor roughness, [33],[34]. The largely geophysical issues associated with this question were dealt with only in passing, since these are not germane to the physics of the problem and its computational aspects.

4) Initial computations were carried out for a variety of acoustic environments and bottom roughness power spectra as a test of the method and its implementation.

3.5 Conclusion

The program described here consisted of the initial stages of the development, implementation, and validation of a three-dimensional finite-element parabolic equation propagation code which will have the capability of scattering acoustic energy from a statistically rough bottom by using an ensemble-average method. This will offer the possibility of accurate modeling in situations where the 2-D approximation is inadequate. Full exploration of these issues will require significant amounts of supercomputer time. Such resources were available at the start of this research, but became unavailable in its later stages. Parallelization is essential, as discussed above, and is being explored.

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